Letters to scientists:

Interference from the future

Thomas Gossmann <thg1aalice.de>
Datum:21 Dez., 2019 12:41AM
An:yokimaumbc.edu
Cc (Kopie):shihaumbc.eduscullyatamu.edulindneramathematik.uni-
kassel.dementemeieramathematik.uni-kassel.de

Thomas Goßmann   Dec18 2019
Kassel, Germany

Interference from the future

Dear Scully, Kim, Shih, Druhl,
Your delayed choice experiment is still mysterious for people, maybe I can help.

1. A little question first:
You stated "Either region A of the BBO or region B emits a photon pair". Isn´t the modern view:
"The photon pair is not emitted by a specific region. Each region emits a possible photon pair, then later when way-measurement happens, a story can be applied where only one of the regions emitted." Just like saying "photon is neither wave nor particle when going through doubleslit" instead of "latter measurement decides if photon is particle or wave when going through"?

2. Interference from the future
Let´s run 50 of Your Delayed Choice Quantum Erasers (1999) simultaneously without coincidence counters, with D3/4,BSA replaced by making BS switchable, with measurement screen instead of scanning, without idler measurement.
I´m not sure about the measurement technique (how far away), I don´t even know how many photons hit the wall around the doubleslit.
Let´s say this is possible, then one would get at t=4ns 50 x-coordinates from signal photons. Then let´s assume all 50 idlers happen to leave BS upwards (where D1 was) at t=20. (all 50 BSon)
Then at t=4 the impacts show interference, right? (not visible well because only 50photons, but visible)
I call this "interference from the future" and try to use it for mathematical retrosignaling proof with standard QM.
Attached 50n simultaneous delayed choices (50 per screen), then if BSon at t=20 one gets at t=4 n screens (signal x-coord) based on probability distributions with additional maxima, compared to BSoff. (rests of interference in the signal soup)
The expected maxima spectrum of screens at t=4 depends on future decision all BSon/ all off?
Then the signal sets BSon/off are distinctable when n big enough, which is one future bit.
volkscomputer.biz/RetrokorrelationFuerMathematiker.pdf
submitted to math profs in Kassel, english complete pictures, please confirm to profs if this makes sense to You.
If measurement technique is good enough, one could do an experiment with coincidence counters to confirm the future bit. Then with better technique experiments without coincidence counters could be done, in search for the time paradox expectation values of QM. (If I see this right.)

3. Dr. Drühl, psycho physics
Copy of this mail messaged to Kai J Druhl at "facebook".

The only reproducible evidence as far as I know is Peoch79 (chicks conditioned to robot influence movement of robot which is caused by random generator), which looks like a new Fundamental Force that influences QM randomness. (which I call "psycho exchange particles")
Does TM help to see the particles? (I guess they do not even perform such simple education.)
For trying to repeat Peoch79 and mentioning Bohm (1991) I was kicked out by feministic-physics local prof. People with more knowledge avoid physicists as long as such simple efforts are not supported. (Hawking on TV inviting time travellers ridiculous)

To Republicans and Christians:
Let´s make a great philosophic connection!
1. international time research group in Kassel:
Combining Peoch79 and delayed choice makes future bit cheap. (best chance)
We start background discussion with You and other relevant groups, in order to prepare for this "end of times as we know it".
The material world is much greater than expected, and connected to non-material world which is real!
2. Oases Union:
volkscomputer.biz/27/27.html
Greta T. failed to ask Pres.T. for 1% of carbon market as suggested.
Support my water-initiative, take part in my bureau in Kassel, to organize the facts and take steps like: giving Siemens a kick because they refused cooperation with oases union and blamed it on US; meeting poor majority on eyelevel, preparing economy in South Europe, North Africa, lithium triangle.
Water to poor people has been controversial because of birth rate.
But now, with Your help, this is a new option. We can bring them desalination technique without debt, if they show responsibility for their environment.
Don´t leave it to the planners. Unlike mathematicians, planners are not aware that partly self-conscious systems are contradictory (Gödel).
3. Philosophy:
Bible is not enough. Important things are missing, some distorted. Church and religious people need extra philosophy, the work is not done. Is this to radical for You?
With the upcoming evolutionary development, things get clearer. (Mental evolution)

So let´s start with little steps and built a bureau for physics and oases union in Kassel.
 I´m just answering to letter from german secretary of state (BMZ) and suggest new calculations for profitability of desalination, which include surrounding effects and future.
If I get order to look closer and report on possible contracts, I found bureau and make plans with inhabitants of dry areas. (Oases kit in preparation, german surety department agreed.)
Then I (we) can accept some donators of fewhundredbucks/month, to prepare mass work.
Maybe someone ask the Bavarian BMZ-Secretary to realize importance, I don´t know exactly what You can do. Maybe You know people over here I can meet.

Best Regards
TG
- - - -

Stochastische Untersuchung Retrokorrelation
Thomas Gossmann <thg1aalice.de>
Datum:11 Dez., 2019 10:24PM
An:lindneramathematik.uni-kassel.de

Thomas Goßmann   11.12.19
Sodensternstr.16
34125 Kassel

Stochastische Untersuchung Retrokorrelation

An die Stochastik-Professoren Uni Kassel, Lindner, Mentemeier,
Sehr geehrte Herren,
anbei eine evtl. hochinteressante Untersuchung über Rückwärts-"Kausalität" in der Physik, auf
aktuellem Niveau: „Retrokorrelation für Mathematiker".

Falls Sie nicht viel von Quantenmechanik wissen, umso besser, es geht um einen rein
mathematischen Aspekt:
Die Würfelmenge (Verteilungswahrscheinlichkeiten) für die Photon-Einschläge wird in der Zukunft
geformt, dort kann man entscheiden, welcher Art die Würfelmenge ist. Laut Standard-
Interpretation der QM darf man die Art der Würfelmenge nicht anhand der Messergebnisse
(Photon-Einschläge) erkennen!
Die Würfelmengen sehen deutlich unterschiedlich aus, daher mein evtl.Verdacht, dass die daraus
folgenden zufälligen Messreihenserien unterschiedliche Maxima-Erwartungswerte haben, sehen
Sie selbst.

Inhalt pdf-Anlage:
4 Seiten exakte mathematische Darlegung für 50n Photonen,
4 Seiten Originalversuch von 1982/2000 für 1 Photon, US-deutsch-russisches Team, gesponsored
von Navy, Army, NSA und Wissenschaftsstiftungen

Wenn Sie den Verdacht teilen (z.B. wenn es für diese Situation keine allgemeinen Sätze gibt, die die
Gleichheit der Maxima-Erwartungskurven erzwingen), muss zunächst nochmal überprüft werden,
ob ich die QM-Situation richtig wiedergegeben habe (ist aber ziemlich sicher). Dann kann ich, ohne
Kosten oder Zeitbelastung für Sie, die Analyse für 50xn (statt mxn) Photonen durchführen.
Im Erfolgsfall gibt's neue Mathematikfelder, Ruhm, Drittmittel und Institute.

Die Rechnungen sehen für mich aus, als könnte sie ein Student durchführen, andererseits ist die
Situation ziemlich trickreich und es hat eine Weile gedauert, bis ich drauf gekommen bin. Der
Standard-Eindruck ist, dass nur eine Signalplörre rauskommt, der man nichts ansieht, aber sehen
Sie es sich selbst an. Einige tausend Leute arbeiten in dem Bereich, hunderte müssten schon
hierauf gekommen sein. Vielleicht hat es noch keiner versucht.
Eine Arbeitsgruppe einer "Universität von Kalifornien" hat mal versucht, die Situation bei
Wikipedia zu erläutern, das war nur peinlich und wenig qualifiziert.

Meine Forschungsgebiete sind Multiplikatoreffekte im Bereich Physik, Ökologie u.a.,
wobei ich keine herkömmliche Universitäts-Karriere habe, sondern mich bei Bedarf einarbeiten
kann.
Wenn hier z.B. stochastische Differentialgleichungen weiterhelfen, kann ich das schnell nachholen.
(Beim Mathestudium in Göttingen übersprungen glaube ich.)

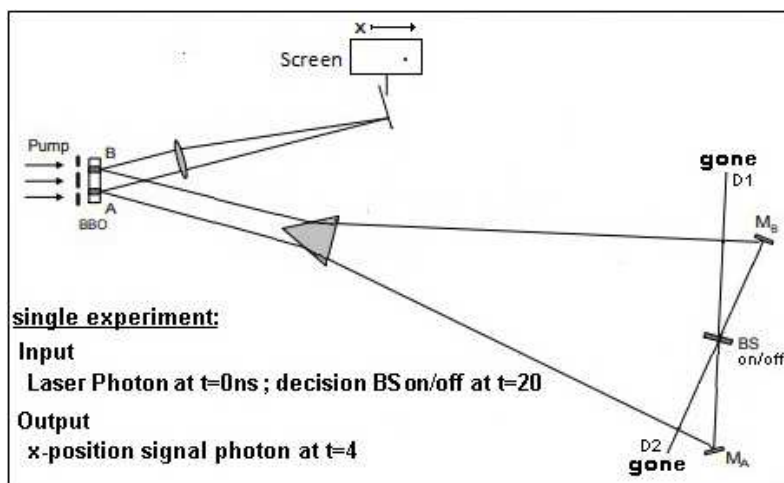Mit freundlichen Grüßen
TG

Retrokorrelation für Mathematiker

Ein Photon trifft auf einen Messschirm, die x-Koordinate entsteht zufällig anhand einer umstandsabhängigen Wahrscheinlichkeitsverteilung, im folgenden genannt Würfel.
Welcher Würfel verwendet wird, entscheidet sich erst später, anhand eines Schalters und eines weiteren Zufalls an einem verschränkten Photon (Reflexion/Durchgang an 50:50-Strahlteiler).
Anhand des Photon-Einschlags kann man nicht erkennen, welcher Würfel verwendet wird.
Dass der frühere Orts-Messzufall am Schirm den späteren 50:50-Zufall verursacht, gilt als ausgeschlossen.
Der Würfel wird also tatsächlich in der Zukunft geformt.
Dabei soll aber keine Information in die Vergangenheit fließen, sondern es handele sich nur um eine Korrelation.
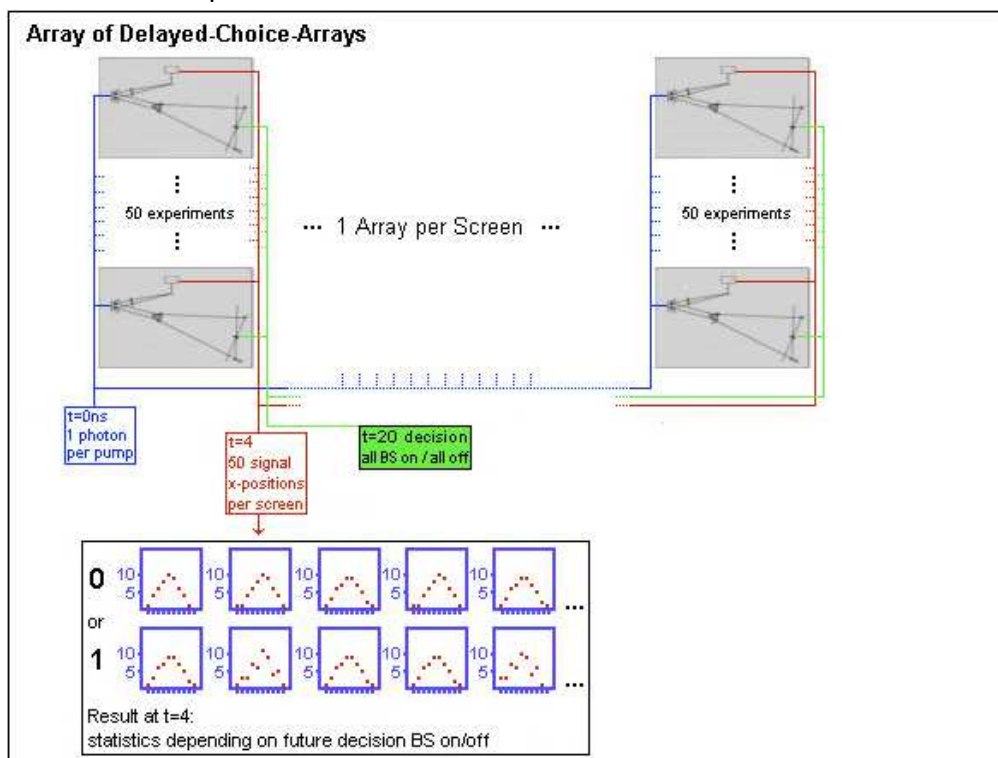Für spezielle Gruppen von Photonen wurde dies aber meines Wissens nicht erwiesen, sondern es wird immer noch über den Informationsfluss spekuliert.
Um dies teilweise oder abschließend zu klären habe ich folgende Forschungs- oder Übungsaufgabe erstellt:

leicht modifiziertes Experiment von Kim...Scully 2000 "Delayed Choice Quantum Eraser"
BSoff:=100%Durchzug als ob BS nicht vorhanden (oder alle BSoff:=100% Reflexion wie 2 Spiegel)
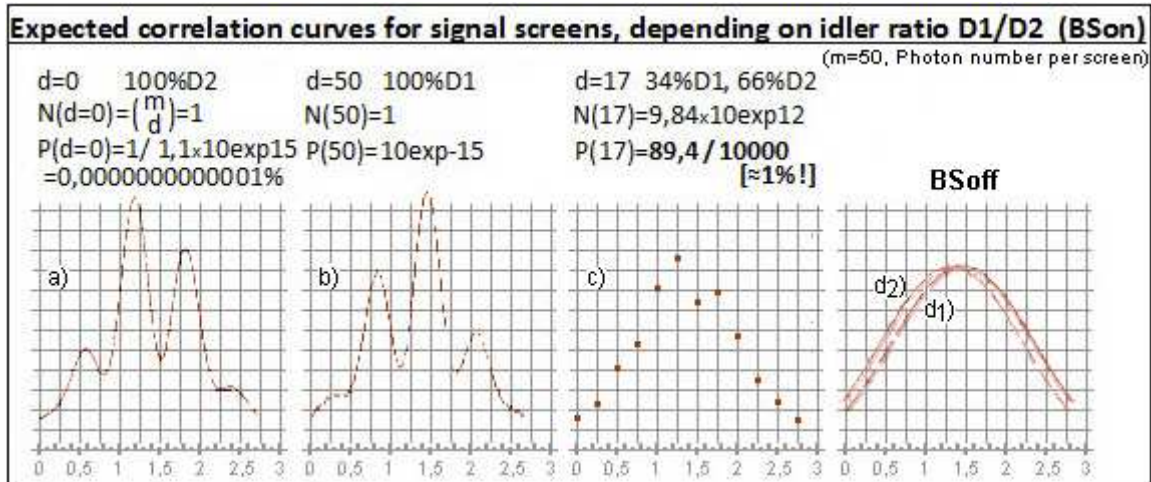
50n simultane Experimente:

Durch die Entscheidung "Strahlteiler an/aus für alle 50n" und die 50n Zufälle "Sekundärphoton geht nach oben/unten" bei t=20 wird eine Menge aus n Würfeln gebildet, die aus zufälligen Mischungen der zwei "reinen Würfel" a),b) (BSon) oder d1),d2) (BSoff) bestehen, bei denen zufällig alle Sekundärphotonen nach oben/unten verschwinden.

Bsp.:

a) Strahlteiler an und alle 50 Sekundärphotonen des Messschirms entweichen zufällig nach unten

b) Strahlteiler an und alle 50 Sekundärphotonen (idler) nach oben

c) Strahlteiler an, 17 nach oben, 33 nach unten

d1) Strahlteiler aus, alle 50 oben, d2) Strahlteiler aus, alle 50 unten

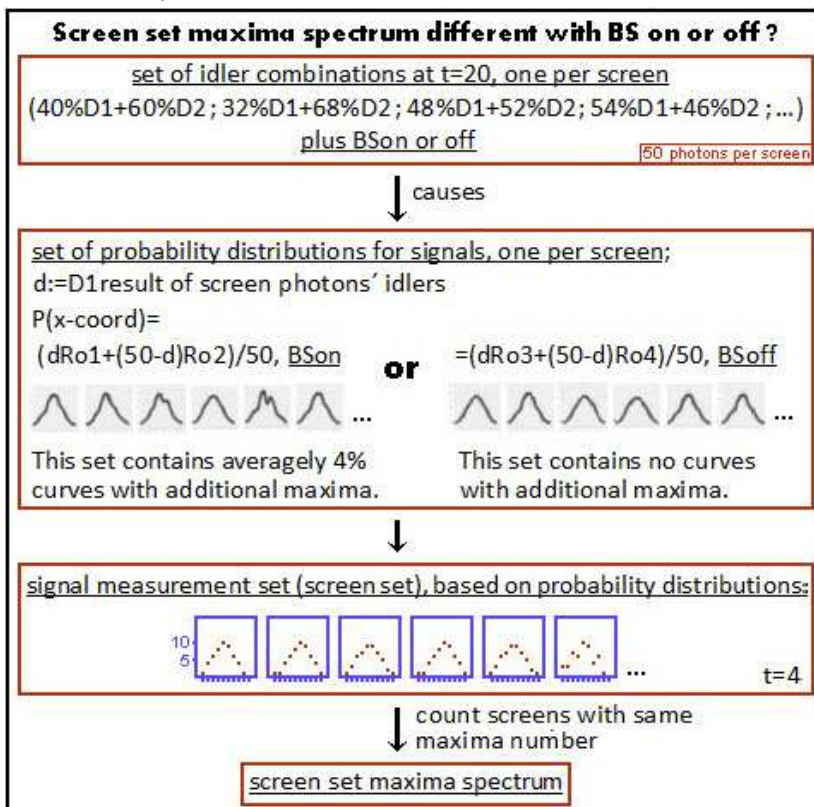(102 mögliche Würfel; mit je 1 Würfel wird 50mal gleichzeitig pro Schirm gewürfelt, n Schirme)

Expected correlation curves for signal screens, depending on idler ratio D1/D2 (BSon)
(m=50, Photon number per screen)

d=0   100%D2   d=50 100%D1   d=17 34%D1, 66%D2
$N(d=0)=\binom{m}{d}=1$ $N(50)=1$ $N(17)=9{,}84\times10^{12}$
$P(d=0)=1/1{,}1\times10^{15}$ $P(50)=10^{-15}$ $P(17)=89{,}4/10000$
$=0{,}0000000000001\%$ $[\approx 1\%!]$ **BSoff**

a)   b)   c)   d2) d1)

0 0,5 1 1,5 2 2,5 3   0 0,5 1 1,5 2 2,5 3   0 0,5 1 1,5 2 2,5 3   0 0,5 1 1,5 2 2,5 3

Roi(x):=Original-Wahrscheinlichkeitsverteilung Kim...Scully i=1,..4 (1 für BSon/oben...4 BSoff/unten)

Würfel (expected correlation curves):

$P_{an}(x)= 1/50\ (dRo1+ (50-d)Ro2)$, BSon , d=Anzahl der Photonen, die nach oben entweichen

$P_{aus}(x)= 1/50\ (dRo3+ (50-d)Ro4)$, BSoff

Laut Interpretation der Quantenphysik dürfen die Messstatistiken "0" und "1" nicht erkennbar unterschiedlich sein. Dazu muss auch der Erwartungswert für Maxima in den Messkurven-Mengen 100%ig identisch sein. Aus den Wahrscheinlichkeitskurven Roi muss daher folgen "Maxima-Erwartungswert identisch", sieht aber nicht so aus:

Screen set maxima spectrum different with BS on or off ?

set of idler combinations at t=20, one per screen
(40%D1+60%D2 ; 32%D1+68%D2 ; 48%D1+52%D2 ; 54%D1+46%D2 ; …)
plus BSon or off
50 photons per screen

↓ causes

set of probability distributions for signals, one per screen;
d:=D1result of screen photons´ idlers
P(x-coord)=
(dRo1+(50-d)Ro2)/50, BSon   **or**   =(dRo3+(50-d)Ro4)/50, BSoff

This set contains averagely 4% curves with additional maxima.   This set contains no curves with additional maxima.

↓

signal measurement set (screen set), based on probability distributions:

t=4

↓ count screens with same maxima number

screen set maxima spectrum

Eine Erhöhung der Photonenzahl pro Schirm verbessert zwar die Ausleuchtung der Maxima, aber die Würfel mit zusätzlichen Maxima werden extrem selten (Zusammensetzung der Würfelmenge abhängig von Photonenzahl pro Schirm).

Die erhöhte Wahrscheinlichkeit für Maxima in der Messkurve bei 4% der Würfel P_{an} (weil diese zusätzliche Maxima enthalten) wird vielleicht dadurch kompensiert, dass viele P_{aus} flachere Abschnitte haben mit erhöhter Wahrscheinlichkeit für Maxima in der Messkurve, sieht auch nicht so aus. (Die Mischungen d1)+d2) sind steiler als d1),d2), und einen allgemeinen Satz für sowas kenne ich nicht.)

Die Funktionsvorschriften für die Erwartungskurven Roi konnte ich noch nicht ermitteln, um einen allgemeinen Beweis Roi => M(P_{an})=M(P_{aus}) anzugehen. (Linse, Doppelspalt, Frequenz an Messschirm anpassen, danebengehende Photonen berücksichtigen wg. Verzicht auf coincidence counters).

Die schlimmen Kombinatorik-Berechnungen für den Fall "50Photonen pro Schirm" könnte ich alleine durchführen, mir fehlt aber Werkzeug und Diskussion.

Ich bitte daher um Vermittlung von Interessenten: Prof., der dies als mögliche Diplomarbeit einordnet (Dr.Arbeit falls Informationsübertragung bewiesen), oder als interessante Übungsaufgabe; Physik-Doktoranten (ich zahle auch für Nachhilfe); Mathematiker mit ähnlichem Interesse; sonstige

Mit freundlichen Grüßen
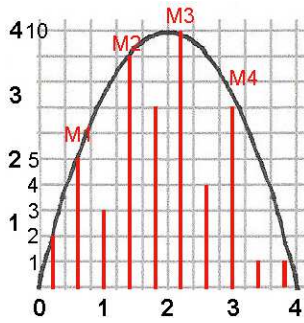Thomas Goßmann, Kassel, 30.11.19
thg1aalice.de
0561 573152

Wissenslücke   einfache Berechnung von Maxima-Erwartungswert in Messkurve

Bsp.

gegeben Wahrscheinlichkeitsverteilung P(x)=-x²+4x, damit wird 50mal gewürfelt in 10 Intervallen:

rot: Bsp.-Messreihe mit 4 Maxima

Wie hoch ist der Erwartungswert für Maxima in der Mess-"Kurve"?

Ich müsste das kombinatorisch durchgehen, $P(Intervall):= 3/32 \; |\int_{x_1}^{x_2} P(x)dx|$, dann gleichgroße „Kästchen" bilden, den Intervallen zuordnen entsprechend der Wahrscheinlichkeit, dann Bälle auf Kästchen verteilen. (Viele Kästchen, um die Kommastellen zu berücksichtigen.)
$\Sigma P(Intervall)=1=100\%$

Gibt´s eine Formel aus der hervorgeht „Mit x% Wahrscheinlichkeit hat die Messreihe 0 Maxima, mit y% 1 Max, mit z% 2Max,..." ?   Var.: P(x)= -x²+4x (von x=0 bis 4), m=50 mal würfeln, I=10 Intervalle. Oder einen Satz wo etwas über P(I max) aus P(I) und P(Nachbarintervalle) folgt, irgendwas. Oder muss man sowas zu Fuß machen...

- - - -

0/1-Erkennung (retrosignaling) im Fall M(BSon)‡M(BSoff)

m=50 Photonen pro Schirm
n Schirme
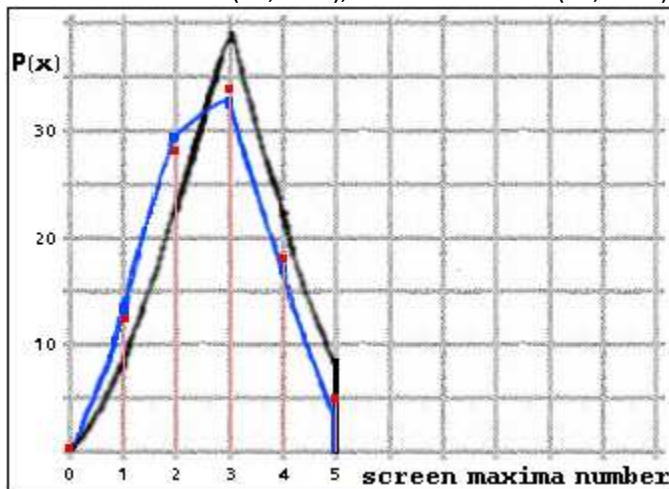M[D1;D2] = M[d; 50-d] := Balkengraph des Würfels D1/D2, Wahrscheinlichkeit über Maximaanzahl

M(50) := [d=0bis50]ΣM[d;50-d](m über d)/2exp50 =
M[0;50]ₓ10exp-15 + M[1;49]ₓ5ₓ10exp-14 + … +M[50;0]ₓ10exp-15
durchschnittlicher Maxima-Erwartungsgraph für m=50, unabhängig von n

schwarz=Hüllkurve M(50,BSon), blau=Hüllkurve M(50,BSoff):

Beispielgraph für M(50,BSon)‡M(50,BSoff). Rote Punkte: Beispielmessreihe für hohe n.
Die Messkurve nähert sich der Erwartungskurve, so dass sie mit 99,x%iger Sicherheit identifizierbar ist als 0 oder 1 (future bit).

A Delayed Choice Quantum Eraser

Yoon-Ho Kim, R. Yu, S.P. Kulik*, and Y.H. Shih
Department of Physics, University of Maryland, Baltimore County,
Baltimore, MD 21250

Marlan O. Scully
Department of Physics, Texas A & M University, College Station, TX 77842
and Max-Planck Institut für Quantenoptik, München, Germany
(submitted to PRL)

This paper reports a "delayed choice quantum eraser" experiment proposed by Scully and Drühl in 1982. The experimental results demonstrated the possibility of simultaneously observing both particle-like and wave-like behavior of a quantum via quantum entanglement. The which-path or both-path information of a quantum can be erased or marked by its entangled twin even after the registration of the quantum.

PACS Number: 03.65.Bz, 42.50.Dv

Complementarity, perhaps the most basic principle of quantum mechanics, distinguishes the world of quantum phenomena from the realm of classical physics. Quantum mechanically, one can never expect to measure both precise position and momentum of a quantum at the same time. It is prohibited. We say that the quantum observables "position" and "momentum" are "complementary" because the precise knowledge of the position (momentum) implies that all possible outcomes of measuring the momentum (position) are equally probable. In 1927, Niels Bohr illustrated complementarity with "wave-like" and "particle-like" attributes of a quantum mechanical object [1]. Since then, complementarity is often superficially identified with "wave-particle duality of matter". Over the years the two-slit interference experiment has been emphasized as a good example of the enforcement of complementarity. Feynman, discussing the two-slit experiment, noted that this wave-particle dual behavior contains the basic mystery of quantum mechanics [2]. The actual mechanisms that enforce complementarity vary from one experimental situation to another. In the two-slit experiment, the common "wisdom" is that the position-momentum uncertainty relation $\delta x \delta p \geq \frac{\hbar}{2}$ makes it impossible to determine which slit the photon (or electron) passes through without at the same time disturbing the photon (or electron) enough to destroy the interference pattern. However, it has been proven [3] that under certain circumstances this common interpretation may not be true. In 1982, Scully and Drühl found a way around this position-momentum uncertainty obstacle and proposed a quantum eraser to obtain which-path or particle-like information without scattering or

otherwise introducing large uncontrolled phase factors to disturb the interference. To be sure the interference pattern disappears when which-path information is obtained. But it reappears when we erase (quantum erasure) the which-path information [3,4]. Since 1982, quantum eraser behavior has been reported in several experiments [5]; however, the original scheme has not been fully demonstrated.

One proposed quantum eraser experiment very close to the 1982 proposal is illustrated in Fig.1. Two atoms labeled by A and B are excited by a laser pulse. A pair of entangled photons, photon 1 and photon 2, is then emitted from either atom A or atom B by atomic cascade decay. Photon 1, propagating to the right, is registered by a photon counting detector D_0, which can be scanned by a step motor along its x-axis for the observation of interference fringes. Photon 2, propagating to the left, is injected into a beamsplitter. If the pair is generated in atom A, photon 2 will follow the A path meeting BSA with 50% chance of being reflected or transmitted. If the pair is generated in atom B, photon 2 will follow the B path meeting BSB with 50% chance of being reflected or transmitted. Under the 50% chance of being transmitted by either BSA or BSB, photon 2 is detected by either detector D_3 or D_4. The registration of D_3 or D_4 provides which-path information (path A or path B) of photon 2 and in turn provides which-path information of photon 1 because of the entanglement nature of the two-photon state of atomic cascade decay. Given a reflection at either BSA or BSB photon 2 will continue to follow its A path or B path to meet another 50-50 beamsplitter BS and then be detected by either detector D_1 or D_2, which are placed at the output ports of the beamsplitter BS. The triggering of detectors D_1 or D_2 erases the which-path information. So that either the absence of the interference or the restoration of the interference can be arranged via an appropriately contrived photon correlation study. The

---

experiment is designed in such a way that L_0, the optical distance between atoms A, B and detector D_0, is much shorter than L_i, which is the optical distance between atoms A, B and detectors D_1, D_2, D_3, and D_4, respectively. So that D_0 will be triggered much earlier by photon 1. After the registration of photon 1, we look at these "delayed" detection events of D_1, D_2, D_3, and D_4 which have constant time delays, $\tau_i \simeq (L_i - L_0)/c$, relative to the triggering time of D_0. It is easy to see these "joint detection" events must have resulted from the same photon pair. It was predicted that the "joint detection" counting rate R_{01} (joint detection rate between D_0 and D_1) and R_{02} will show interference pattern when detector D_0 is scanned along its x-axis. This reflects the wave property (both-path) of photon 1. However, no interference will be observed in the "joint detection" counting rate R_{03} and R_{04} when detector D_0 is scanned along its x-axis. This is clearly expected because we now have indicated the particle property (which-path) of photon 1. It is important to emphasize that all four "joint detection" rates R_{01}, R_{02}, R_{03}, and R_{04} are recorded at the same time during one scanning of D_0 along its y-axis. That is, in the present experiment we "see" both wave (interference) and which-path (particle-like) with the same apparatus.

We wish to report a realization of the above quantum eraser experiment. The schematic diagram of the experimental setup is shown in Fig.2. Instead of atomic cascade decay, spontaneous parametric down conversion (SPDC) is used to prepare the entangled two-photon state. SPDC is a spontaneous nonlinear optical process from which a pair of signal-idler photons is generated when a pump laser beam is incident onto a nonlinear optical crystal [6]. In this experiment, the $351.1nm$ Argon ion pump laser beam is divided by a double-slit and incident onto a type-II phase matching [7] nonlinear optical crystal BBO ($\beta - BaB_2O_4$) at two regions A and B. A pair of $702.2nm$ orthogonally polarized signal-idler photon is generated either from A or B region. The width of the SPDC region is about $0.3mm$ and the distance between the center of A and B is about $0.7mm$. A Glen-Thompson prism is used to split the orthogonally polarized signal and idler. The signal photon (photon 1, either from A or B) passes a lens LS to meet detector D_0, which is placed on the Fourier transform plane (focal plane for collimated light beam) of the lens. The use of lens LS is to achieve the "far field" condition, but still keep a short distance between the slit and the detector D_0. Detector D_0 can be scanned along its x-axis by a step motor. The idler photon (photon 2) is sent to an interferometer with equal-path optical arms. The interferometer includes a prism PS, two 50-50 beamsplitters BSA, BSB, two reflecting mirrors M_A, M_B, and a 50-50 beamsplitter BS. Detectors D_1 and D_2 are placed at the two output ports of the BS, respectively, for erasing the which-path information. The triggering of detectors D_3 and D_4 provide which-path information of the idler (photon 2) and in turn provide which-path information of the signal (photon 1). The electronic output pulses of detectors D_1, D_2, D_3, and D_4 are sent to coincidence circuits with the output pulse of detector D_0, respectively, for the counting of "joint detection" rates R_{01}, R_{02}, R_{03}, and R_{04}. In this experiment the optical delay $(L_i - L_0)$ is chosen to be $\simeq 2.5m$, where L_0 is the optical distance between the output surface of BBO and detector D_0, and L_i is the optical distance between the output surface of the BBO and detectors D_1, D_2, D_3, and D_4, respectively. This means that any information one can learn from photon 2 must be at least $8ns$ later than what one has learned from the registration of photon 1. Compared to the $1ns$ response time of the detectors, $2.5m$ delay is good enough for a "delayed erasure".

Figs.3, 4, and 5 report the experimental results, which are all consistent with prediction. Figs.3 and 4 show the "joint detection" rates R_{01} and R_{02} against the x coordinates of detector D_0. It is clear we have observed the standard Young's double-slit interference pattern. However, there is a π phase shift between the two interference fringes. The π phase shift is explained as follows. Fig.5 reports a typical R_{03} (R_{04}), "joint detection" counting rate between D_0 and "which-path" D_3 (D_4), against the x coordinates of detector D_0. An absence of interference is clearly demonstrated. There is no significant difference between the curves of R_{03} and R_{04} except the small shift of the center.

To explain the experimental results, a standard quantum mechanical calculation is presented in the following. The "joint detection" counting rate, R_{0i}, of detector D_0 and detector D_j, on the time interval T, is given by the Glauber formula [8]:

$$R_{0j} \propto \frac{1}{T} \int_0^T \int_0^T dT_0 dT_j \langle \Psi | E_0^{(-)} E_j^{(-)} E_j^{(+)} E_0^{(+)} | \Psi \rangle$$
$$= \frac{1}{T} \int_0^T \int_0^T dT_0 dT_j |\langle 0 | E_j^{(+)} E_0^{(+)} | \Psi \rangle|^2, \qquad (1)$$

where T_0 is the detection time of D_0, T_j is the detection time of D_j ($j = 1, 2, 3, 4$) and $E_{0,j}^{(\pm)}$ are positive and negative-frequency components of the field at detectors D_0 and D_j, respectively. $|\Psi\rangle$ is the entangled state of SPDC,

$$|\Psi\rangle = \sum_{s,i} C(\mathbf{k}_s, \mathbf{k}_i) \, a_s^\dagger(\omega(\mathbf{k}_s)) \, a_i^\dagger(\omega(\mathbf{k}_i))|0\rangle, \qquad (2)$$

where $C(\mathbf{k}_s, \mathbf{k}_i) = \delta(\omega_s + \omega_i - \omega_p)\delta(\mathbf{k}_s + \mathbf{k}_i - \mathbf{k}_p)$, for the SPDC in which ω_j and $\mathbf{k_j}$ ($j = s, i, p$) are the frequency and wavevectors of the signal (s), idler (i), and pump (p), respectively, ω_p and $\mathbf{k}_p$ can be considered as constants, a single mode laser line is used for pump and $a_s^\dagger$ and $a_i^\dagger$ are creation operators for signal and idler photons, respectively. For the case of two scattering atoms, see ref. [3], and in the case of cascade radiation, see ref. [9], $C(\mathbf{k}_s, \mathbf{k}_i)$ has a similar structure but without the momentum delta function. The δ functions in eq.(2) are the results of approximations for an infinite size SPDC crystal and for infinite interaction time. We introduce the two-dimensional function $\Psi(t_0, t_j)$ as in eq.(1),

$$\Psi(t_0, t_j) \equiv \langle 0 | E_j^{(+)} E_0^{(+)} | \Psi \rangle. \qquad (3)$$

$\Psi(t_0, t_j)$ is the joint count probability amplitude ("wavefunction" for short), where $t_0 \equiv T_0 - L_0/c$, $t_j \equiv T_j - L_j/c$, $j = 1, 2, 3, 4$, L_0 (L_j) is the optical distance between the output point on the BBO crystal and D_0 (D_j). It is straightforward to see that the four "wavefunctions" $\Psi(t_0, t_j)$, correspond to four different "joint detection" measurements, having the following different forms:

$$\Psi(t_0, t_1) = A(t_0, t_1^A) + A(t_0, t_1^B),$$
$$\Psi(t_0, t_2) = A(t_0, t_2^A) - A(t_0, t_2^B), \qquad (4)$$
$$\Psi(t_0, t_3) = A(t_0, t_3^A), \quad \Psi(t_0, t_4) = A(t_0, t_4^B), \qquad (5)$$

where as in Fig.1 the upper index of t (A or B) labels the scattering crystal (A or B region) and the lower index of t indicates different detectors. The different sign between the two amplitudes $\Psi(t_0, t_1)$ and $\Psi(t_0, t_2)$ is caused by the transmission-reflection unitary transformation of the beamsplitter BS, see Fig.1 and Fig.2. It is also straightforward to calculate each of the $A(t_i, t_j)$ [10]. To simplify the calculations, we consider the longitudinal integral only and write the two-photon state in terms of the integral of k_e and k_o:

$$|\Psi\rangle = A_0' \int dk_e \int dk_o \; \delta(\omega_e + \omega_o - \omega_p) \times$$
$$\Phi(\Delta_k L) a_{k_e}^\dagger a_{k_o}^\dagger |0\rangle, \qquad (6)$$

where a type-II phase matching crystal with finite length of L is assumed. $\Phi(\Delta_k L)$ is a sinc-like function, $\Phi(\Delta_k L) = (e^{i(\Delta_k L)} - 1)/i(\Delta_k L)$. Using eqs. (3) and (6) we find,

$$A(t_i, t_j) = A_0 \int dk_e \int dk_o \delta(\omega_e + \omega_o - \omega_p) \times$$
$$\Phi(\Delta_k L) f_i(\omega_e) f_j(\omega_o) e^{-i(\omega_e t_1^e + \omega_o t_2^o)}, \qquad (7)$$

where $f_{i,j}(\omega)$, is the spectral transmission function of an assumed filter placed in front of the k_{th} detector and is assumed Gaussian to simplify the calculation. To complete the integral, we define $\omega_e = \Omega_e + \nu$ and $\omega_o = \Omega_o - \nu$, where Ω_e and Ω_o are the center frequencies of the SPDC, $\Omega_e + \Omega_o = \Omega_p$ and ν is a small tuning frequency, so that $\omega_e + \omega_o = \Omega_p$ still holds. Consequently, we can expand k_e and k_o around $K_e(\Omega_e)$ and $K_o(\Omega_o)$ to first order in ν:

$$k_e = K_e + \nu \left. \frac{d\omega_e}{dk_e} \right|_{\Omega_e} = K_e + \frac{\nu}{u_e},$$
$$k_o = K_o - \nu \left. \frac{d\omega_o}{dk_o} \right|_{\Omega_o} = K_o - \frac{\nu}{u_o}, \qquad (8)$$

where u_e and u_o are recognized as the group velocities of the e-ray and o-ray at frequencies Ω_e and Ω_o, respectively. Completing the integral, the biphoton wavepacket of type-II SPDC is thus:

$$A(t_i, t_j) = A_0 \Pi(t_i - t_j) e^{-i\Omega_i t_i} e^{-i\Omega_j t_j}, \qquad (9)$$

where we have dropped the e, o indices. The shape of $\Pi(t_1 - t_2)$ is determined by the bandwidth of the spectral filters and the parameter DL of the SPDC crystal, where $D \equiv 1/u_o - 1/u_e$. If the filters are removed or have large enough bandwidth, we have a rectangular pulse function $\Pi(t_1 - t_2)$.

$$\Pi(t_0 - t_j) = \begin{cases} 1 & \text{if } 0 \le t_0 - t_j \le DL, \\ 0 & \text{otherwise.} \end{cases}$$

It is easy to find that the two amplitudes in $\Psi(t_0, t_1)$ and $\Psi(t_0, t_2)$ are indistinguishable (overlap in both $t_0 - t_j$ and $t_0 + t_j$), respectively, so that interference is expected in both the coincidence counting rates, R_{01} and R_{02}; however, with a π phase shift due to the different sign,

$$R_{01} \propto \cos^2(x\pi d/\lambda f), \quad \text{and} \quad R_{02} \propto \sin^2(x\pi d/\lambda f).$$

If we consider "slit" A and B both have finite width (not infinitely narrow), an integral is necessary to sum all possible amplitudes along slit A and slit B. We will have a standard interference-diffraction pattern for R_{01} and R_{02},

$$R_{01} \propto \text{sinc}^2(x\pi a/\lambda f) \cos^2(x\pi d/\lambda f),$$
$$R_{02} \propto \text{sinc}^2(x\pi a/\lambda f) \sin^2(x\pi d/\lambda f), \qquad (10)$$

where a is the width of the slit A and B (equal width), d is the distance between the center of slit A and B, $\lambda = \lambda_s = \lambda_i$ is the wavelength of the signal and idler, and f is the focal length of lens LS. We have also applied the "far field approximation" for the signal and equal optical distance of the interferometer for the idler. After considering the finite size of the detectors and the divergence of the pump beam for further integrals, the interference visibility is reduced to the level close to the observation.

For the "joint detection" R_{03} and R_{04}, it is seen that the "wavefunction" in eq.(5) (which clearly provides "which-path" information) has only one amplitude and no interference is expected.

In conclusion, we have realized a quantum eraser experiment of the type proposed in ref. [3]. The experimental results demonstrate the possibility of observing both particle-like and wave-like behavior of a light quantum via quantum mechanical entanglement. The which-path or both-path information of a quantum can be erased or marked by its entangled twin even after the registration of the quantum.

[1] N. Bohr, Naturwissenschaften, **16**, 245 (1928).

[2] R. Feynman, R. Leighton, and M. Sands, *The Feynman Lectures on Physics*, Vol. III, Addison Wesley, Reading (1965).

[3] M.O. Scully and K. Drühl, Phys. Rev. A **25**, 2208 (1982).

[4] See Wheeler's "delayed choice", in *Quantum Theory and Measurement*, edited by J.A. Wheeler and W.H. Zurek, Princeton Univ. Press (1983).

[5] A.G. Zajonc *et al.*, Nature, **353**, 507 (1991); P.G. Kwiat *et al.*, Phys. Rev. A **49**, 61 (1994); T.J. Herzog *et al.*, Phys. Rev. Lett., **75**, 3034 (1995); T.B. Pittman *et al.*, Phys. Rev. Lett., **77**, 1917 (1996).

[6] D.N. Klyshko, *Photon and Nonlinear Optics*, Gordon and Breach Science, New York (1988); A. Yariv, *Quantum Electronics*, John Wiley and Sons, New York (1989).

[7] In type-I SPDC, signal and idler are both ordinary rays of the crystal; however, in type-II SPDC the signal and idler are orthogonal polarized, i.e., one is ordinary ray and the other is extraordinary ray of the crystal.

[8] R.J. Glauber, Phys. Rev. **130**, 2529 (1963); **131**, 2766 (1963).

[9] M.O. Scully and M.S. Zubairy, *Quantum Optics*, Cambridge Univ. Press, Cambridge, UK (1997).

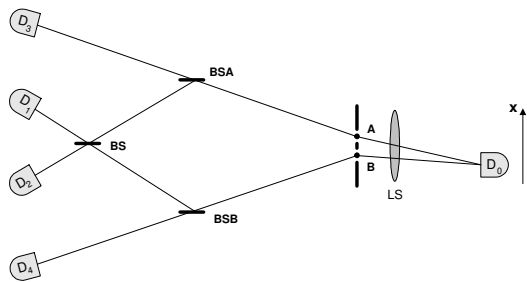[10] M.H. Rubin, D.N. Klyshko, and Y.H. Shih, Phys. Rev. A **50**, 5122 (1994).

FIG. 1. A proposed quantum eraser experiment. A pair of entangled photons is emitted from either atom A or atom B by atomic cascade decay. "Clicks" at D_3 or D_4 provide which-path information and "clicks" at D_1 or D_2 erase the which-path information.
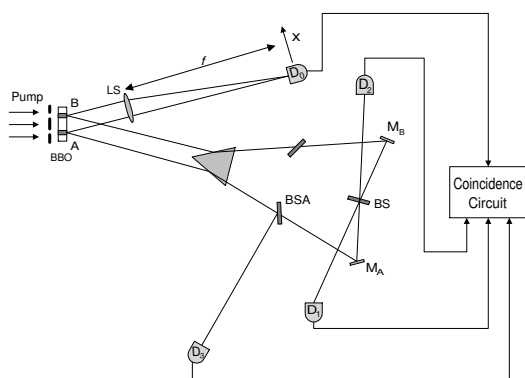

FIG. 2. Schematic of the experimental setup. The pump laser beam of SPDC is divided by a double-slit and incident onto a BBO crystal at two regions A and B. A pair of signal-idler photons is generated either from A or B region. The detection time of the signal photon is $8ns$ earlier than that of the idler.
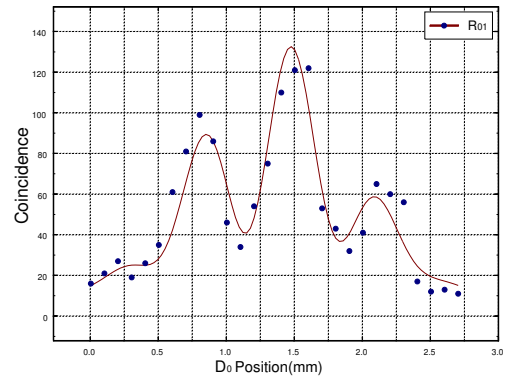

FIG. 3. R_{01} ("joint detection" rate between detectors D_0 and D_1) against the x coordinates of detector D_0. A standard Young's double-slit interference pattern is observed.
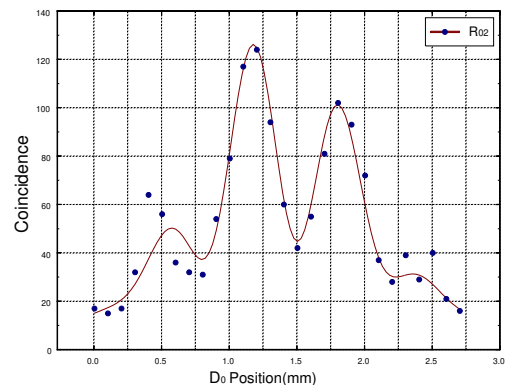

FIG. 4. R_{02} ("joint detection" rate between detectors D_0 and D_2) Note, there is a π phase shift compare to R_{01} shown in Fig.3
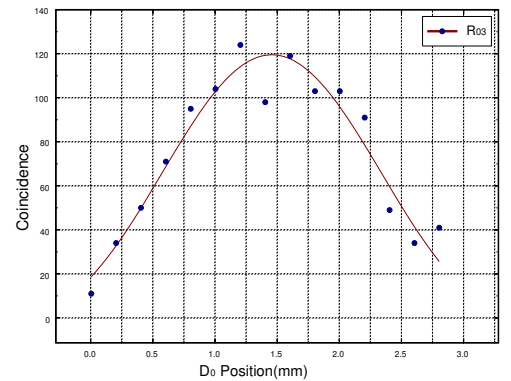

FIG. 5. R_{03} ("joint detection" rate between detectors D_0 and D_3). An absence of interference is clearly demonstrated.